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COMPARISON OF DIFFERENT INVERSION TECHNIQUES FOR DETERMINING PHYSICAL PARAMETERS OF POROUS MEDIA

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ABSTRACT

In modelling the acoustical behaviour of porous materials, the determination of the physical parameters is a fundamental issue. Because of the difficulties in measuring directly some of these quantities, it is possible to use inverse strategies for calculating them once some acoustical parameters are experimentally known.
In this paper different inverse approaches (i.e. analytical, indirect, iterative and genetic) are presented for a representative range of porous media. They are based on the minimization of the experimental characteristic or surface acoustical properties with respect to the same quantities, determined by applying a theoretical prediction model (chiefly the 5 parameter model proposed by Johnson, Champoux and Allard).

INTRODUCTION

The determination of physical parameters of porous media has became of great significance for predicting the acoustical behaviour of these materials. Some of these quantities (airflow resistivity, open porosity, tortuosity and viscous and thermal characteristic lengths) have been chosen by numerous authors to model the processes taking place within porous media.
Direct measurement of all parameters requires a set of test-rigs to be used and could be however complicated. As an alternative, inverse identification methods can be used to evaluate these parameters from acoustical measurements in a standing wave tube.
In the present work four different inverse methods for determining the physical parameters of porous materials will be investigated. Furthermore details related to the accuracy and reliability of the inversely determined parameters are also reported and discussed.

THE JOHNSON-CHAMPOUX-ALLARD MODEL

The use of Biot theory [1] makes it possible to describe the acoustical behaviour of porous materials. At the same time, in many situations when a material sample is excited by acoustical waves, the frame of this material behaves approximately as acoustically rigid (motionless) over a wide range of frequencies. In this case, the porous material can be replaced on a macroscopic scale by an equivalent fluid of effective density ρ_{eq} and effective bulk modulus K_{eq}. The motionless frame condition can occur either because of high density or elasticity modulus, or because of particular test conditions (i.e. material put on a rigid wall). It is shown that viscous effects and thermal exchanges can be treated separately.
In the widely used equivalent fluid model of Johnson-Champoux-Allard, these effective quantities depend on five macroscopic parameters: the flow resistivity σ, the porosity ϕ, the tortuosity α_∞, and the viscous Λ and thermal Λ' characteristic lengths. According to this model, expressions for the ρ_{eq} and K_{eq} are proposed as:

$$\rho_{eq} = \frac{\alpha_\infty \rho_0}{\phi} + \frac{\sigma}{i\omega}\sqrt{1 + \frac{4i\alpha_\infty^{\ 2}\eta\rho_0\omega}{\sigma^2\Lambda^2\phi^2}} \quad \text{and} \quad K_{eq} = \frac{\kappa \cdot P_0/\phi}{\kappa - (\kappa - 1)\left[1 + \frac{8\eta}{i\rho_0\omega N_p\Lambda'^2}\sqrt{1 + \frac{i\rho_0\omega N_p\Lambda'^2}{16\eta}}\right]^{-1}} \quad \text{(Eq. 1)}$$

with ρ_0 and η the density and the viscosity of air, N_p the Prandtl number, κ the specific heat ratio and P_0 the static pressure.
All the acoustical parameters (characteristic impedance and complex wave number) can be deduced from these complex parameters, with:

$$Z_c = \sqrt{\rho_{eq} \cdot K_{eq}} \quad \text{[Ns/m}^3\text{]} \quad \text{and} \quad k_c = \omega\sqrt{\rho_{eq}/K_{eq}} \quad \text{[m}^{-1}\text{]} \qquad \text{(Eq. 2)}$$

from which is possible to calculate the surface impedance by using the following expression:

$$Z_S = Z_c \cdot \cot(k_c \cdot l) \quad \text{[Ns/m}^3\text{]} \qquad \text{(Eq. 3)}$$

when l is the thickness of the material.

DESCRIPTIONS OF THE INVERSE METHODS

Analytical methods

This category is based on the limit behaviour of the bulk properties (effective density and effective bulk modulus).
For determining airflow resistivity the method worked out in [2] here is used. Decomposing the effective density (Eq.1) in real and imaginary part, it is possible to demonstrate that:

$$\sigma = -\lim_{\omega \to 0}\left(Im\left\{\rho_{eq}\right\} \cdot \omega \right) \qquad \text{(Eq. 4)}$$

The method for determining tortuosity is based on the ratio of velocity within the material to velocity in air. In fact, by manipulating the expressions (Eq.1) and (Eq.2) to calculate the complex velocity c it can be proved that:

$$\alpha_\infty = \left\{ \lim_{\omega^{-1/2} \to 0} \left(c/c_0 \right) \right\}^{-2} \qquad \text{(Eq. 5)}$$

An approach for determining the value of porosity can be deduced from the method presented in [2]. In fact the real high frequency limit of the effective density ρ_{eq} gives:

$$\phi = \rho_0 \alpha_\infty \Big/ \left(\lim_{\omega \to \infty} Re\left\{\rho_{eq}\right\} \right) \qquad \text{(Eq. 6)}$$

For the determination of viscous characteristic length Λ, again a method proposed in [2] is utilized. By using the real and imaginary parts of ρ_{eq} , it is possible to demonstrate that:

$$\Lambda = \lim_{\omega \to \infty}\left(\alpha_\infty \sqrt{\frac{2\rho_0 \eta}{\omega\phi\, Im\left\{\rho_{eq}\right\}\left(\rho_0\alpha_\infty - \phi\, Re\left\{\rho_{eq}\right\}\right)}} \right) \qquad \text{(Eq. 7)}$$

Finally once the viscous characteristic length is known it is possible to use an approach equivalent to the $Q\delta$ method, described in [3], for determining Λ' as follow:

$$\Lambda' = \left[\frac{\sqrt{N_{Pr}}}{\kappa - 1} \left(\lim_{\omega \to \infty} \left\{ \frac{Re\left\{kc\right\}}{Im\left\{kc\right\}} \sqrt{2\eta/\omega\rho_0} \right\} - \frac{1}{\Lambda} \right) \right]^{-1} \qquad \text{(Eq. 8)}$$

The quantities in (Eq.6) to (Eq.8) are frequency dependent; their estimation is obtained calculating their mean value in the high frequency range.

Indirect methods

These techniques make use of non linear best-fit algorithms (Nelder-Mead Simplex Method) to determine the best solution to minimize a cost function (bulk property or equivalently surface parameter) calculated by means of a prediction model. In this analysis the amplitude of the normalized surface impedance has been used as cost function:

$$CF\left\{|Zs|\right\} = \sum \left| Z_{S\,meas} - Z_{S\,\mathrm{model}} \right| \qquad \text{(Eq. 9)}$$

The experimental normalized surface impedance is measured by means of Transfer Function technique or equivalently determined with the characteristic impedance and the complex wavenumber, measured using the Transfer Matrix approach. About the model, (Eq. 3) was used to determine the surface impedance once the complex acoustical properties were known through (Eq.1) and (Eq.2).

Genetic algorithm based methods

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution.
For this analysis the same cost function (Eq. 9) has been used and the procedure has been implemented in Matlab®.

Iterative methods

This typology of inversion technique is similar to the indirect and genetic algorithm method in the sense that a cost function is to be minimized with respect to a prediction model. However the algorithm is different since minimization is obtained with continuous iterative change of the physical parameters. For this analysis the same cost function (Eq. 9) has been investigated.

Regarding the last three typologies of inverse techniques, the analysis was carried out in the frequency range between 150 and 4200 Hz. The bounds used for all the tested materials are reported in Table 1.

Table 1. Bounds for the indirect method

	σ	ϕ	α_∞	Λ [μm]	Λ' [μm]
Lower limit	1000	0.1	1	10	10
Upper limit	200000	1	10	2000	2000

In addition, in the case of genetic algorithm it has been applied the non linear constrain $\Lambda' \geq \Lambda$, physically expected.

RESULTS

In this section the inverse methods described above are applied to real porous materials.
The experimental tests have been carried out on 10 porous materials (Polyurethane Foam, Melamine Foam, Glass Wool, Felt, Hemp Fiber, Cellular Rubber) with density between 10 and 78 kg/m^3 and thickness between 14 and 40 mm.
Firstly some non acoustical parameters have been experimentally determined. In particular, the airflow resistivity was measured according to ISO Standard 9053 [4] with the alternate flow method. The open porosity was measured through a method based on Boyle's Law, using isothermal compression of air volume within and external to the tested material [5]. Finally, the tortuosity was determined through a method based on determination of the high frequency limit for the complex phase velocity within the material [6]. For some materials, the characteristic lengths were also collected from previous researches.
Afterwards the surface properties (i.e. surface impedance and the normal incidence sound absorption coefficient) were measured according to the ISO Standard 10354-2 [7]. Lastly, the complex acoustical parameters (i.e. characteristic impedance and complex wave number) were determined by means of a transfer matrix approach by using a 3 microphone technique [8].

The physical and acoustical parameters have been measured on samples of diameter equal to 45mm.
The comparison between the measured parameters and their inversely determined values are reported in Table 2.

Table 2. Comparison between measured and inversely determined parameters

	σ	ϕ	α_∞	Λ [μm]	Λ' [μm]	σ	ϕ	α_∞	Λ [μm]	Λ' [μm]
	M1: Polyurethane Foam 29 Kg/m^3 - 40mm					**M2:** Polyurethane Foam 25 Kg/m^3 - 20mm				
Measured	**5359**	**1.00**	**1.08**	**--**	**--**	**12901**	**0.98**	**1.41**	**--**	**--**
Analytical	6641	0.90	1.03	83	267	18484	0.81	2.72	170	48
Indirect	6414	0.99	1.15	135	250	5000	1.00	1.00	12	257
Genetic	6252	0.99	1.14	132	251	31728	1.00	1.59	31	648
Iterative	6200	0.99	1.10	130	250	31700	0.99	1.59	32	647
	M3: Polyurethane Foam 55 Kg/m^3 - 20mm					**M4:** Polyurethane Foam 36 Kg/m^3 - 20mm				
Measured	**6412**	**0.98**	**1.18**			**7415**	**0.97**	**1.18**		
Analytical	6854	1.04	2.06	460	61	8188	1.00	1.82	447	62
Indirect	7484	1.00	1.52	108	244	7309	1.00	1.47	121	234
Genetic	7300	1.00	1.52	103	252	5203	0.98	1.43	111	225
Iterative	7300	1.00	1.50	100	250	5200	0.98	1.40	110	220
	M5: Polyurethane Foam 30 Kg/m^3 - 30mm					**M6:** Melamine Foam 10 Kg/m^3 - 20mm				
Measured	**5503**	**0.99**	**1.08**	**--**	**--**	**10551**	**1.00**	**1.01**	**160**	**290**
Analytical	6414	0.93	1.39	367	65	6414	0.69	1.03	362	78
Indirect	5305	1.00	1.16	157	254	18316	1.00	1.28	322	346
Genetic	4155	0.98	1.12	141	244	14723	0.98	1.10	92	258
Iterative	4100	0.98	1.10	140	240	14700	0.98	1.00	90	250
	M7: Glass Wool 17 Kg/m^3 - 20mm					**M8:** Felt 78 Kg/m^3 - 14mm				
Measured	**14187**	**0.99**	**1.00**	**182**	**400**	**73364**	**0.97**	**1.01**	**31**	**62**
Analytical	16685	0.79	1.09	195	26	69655	0.74	1.39	134	16
Indirect	26068	0.92	1.00	79	127	81716	1.00	1.00	17	100
Genetic	28055	1.00	1.00	67	190	77697	0.95	1.00	29	113
Iterative	28000	1.00	1.00	60	180	77700	0.95	1.00	20	110
	M9: Hemp fiber 51 Kg/m^3 - 20mm					**M10:** Cellular Rubber 68 Kg/m^3 - 24mm				
Measured	**6215**	**0.99**	**1.05**	**--**	**--**	**123502**	**0.83**	**2.64**	**9**	**15**
Analytical	6488	0.83	1.28	348	47	89401	0.94	4.95	156	29
Indirect	9570	1.00	1.05	68	180	115900	0.81	1.00	8	10
Genetic	9561	1.00	1.05	67	179	121700	0.86	1.29	10	10
Iterative	9560	1.00	1.00	60	170	121690	0.86	1.20	10	10

The previous data can be compared in terms of relative error, that is calculated as:

$$E\% = \left| \frac{Expected - Calculated}{Expected} \right| \quad \text{(Eq. 10)}$$

The average values of the above mentioned quantity for all the tested material, for each physical parameter are reported in Table 3.

Table 3. Average values of the relative errors

	σ	ϕ	α_∞	Λ [μm]	Λ' [μm]
Analytical	19%	15%	41%	525%	83%
Indirect	38%	2%	19%	52%	46%
Genetic	48%	1%	13%	31%	45%
Iterative	48%	1%	12%	39%	45%

Regarding the airflow resistivity, it can be observed that in almost all the cases the analytical method gives a reliable estimate of such quantity. For all the other parameters both genetic and iterative methods provide the best results. Indirect methods provides reliable results for all the physical parameters, although solutions could correspond to local minima.
It is important to remark that the reliability of iterative method depends mainly on the iteration steps. It is worth noting that this procedure is extremely time consuming. For example if we consider the intervals in Table 1 and steps equal to 500, 0.01, 0.01, 10, and 10 for the airflow resistivity, porosity, tortuosity, VCL and TCL respectively, the entire iterative procedure requires 1.49E11 iterations, which means few days for a 2.4 GHz PC.
Furthermore the analytical method could lead to incorrect values of the parameter mainly due to

a not sufficient high frequency measurement range and to the difficulty in finding an adequate range for the linear interpolation. In particular this could happen if frame resonances occur in the acoustical range of frequencies.
Finally, in order to evaluate the effect of all the solutions on the acoustical data, the curves for the sound absorption coefficient were compared to the experimental values and variations were analysed in term of the function Δ (Table 4), that is the average of the difference between the sound absorption coefficient measured experimentally and determined with the JCA model, using the inversely determined solutions for each method. In the same table the average values are also reported.

Table 4. Average of the differences between the sound absorption coefficient measured experimentally and determined with the model

	M1	**M2**	**M3**	**M4**	**M5**	**M6**	**M7**	**M8**	**M9**	**M10**	**Avg**
Δ Analytical	0.04	0.12	0.06	0.05	0.03	0.12	0.14	0.06	0.20	0.10	**0.091**
Δ Indirect	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.06	0.00	0.01	**0.018**
Δ Genetic	0.01	0.07	0.01	0.01	0.01	0.01	0.01	0.02	0.00	0.02	**0.018**
Δ Iterative	0.01	0.07	0.02	0.01	0.01	0.02	0.02	0.04	0.02	0.02	**0.024**

The table shows that in the majority of cases, the Indirect, Genetic and Iterative methods provide satisfactory estimation of the sound absorption coefficient. On the contrary, analytical methods may lead to an incorrect estimation of the curves. This confirms what was underlined previously, that is these methods do not perform minimizations with respect to surface properties.
In Figure 1 the curves of the real and imaginary part of the normalized surface impedance are shown for the material M1. In Figure 2 the experimental and theoretical curves of the sound absorption coefficient are depicted for the same material.
The figures confirm that the Indirect, Genetic and Iterative methods are reliable schemes for determining the best solutions, providing a good estimation of surface properties.

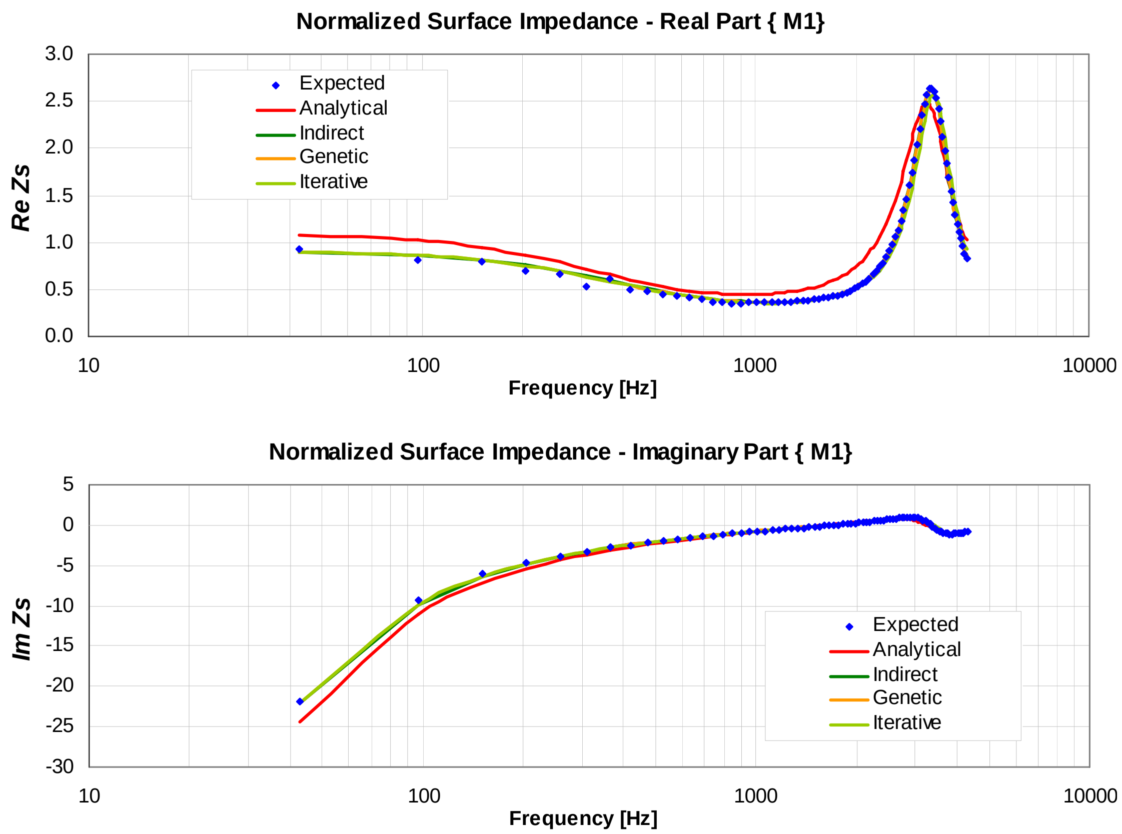

Figure 1. Comparison between experimental and predicted real and imaginary part of the Normalized Surface Impedance for material M1

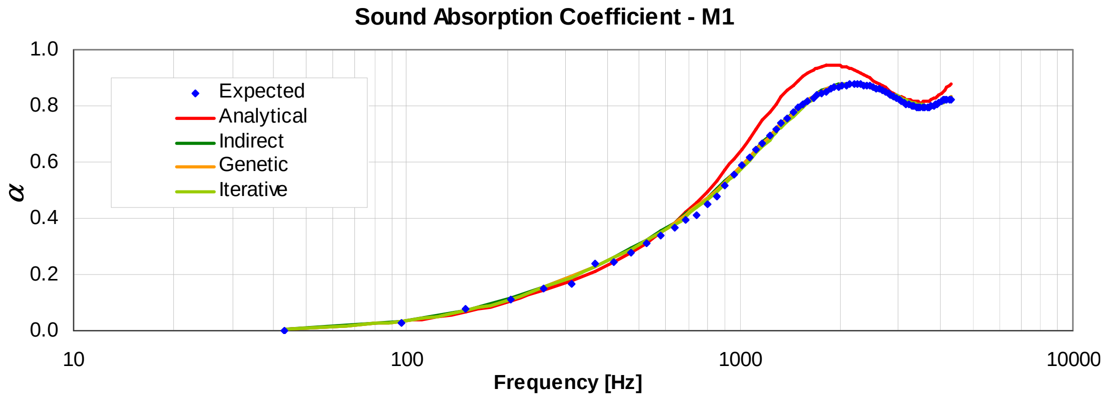

Figure 2. Comparison between experimental and predicted
Sound Absorption Coefficient for material M1

CONCLUSIONS

In this paper different inverse methods for determining the physical parameters of porous materials have been investigated. The main advantages and drawbacks of these methods can be summarized as follow.

Method	Advantages	Disadvantages
Analytical	• Quick estimation; • Minimization of the intrinsic properties of the material, directly correlated to the model.	• Necessity to find adequate ranges for interpolations; • No minimization with respect to surface properties. • Impossibility of implementing non linear constrains.
Indirect	• Quick estimation; • Minimization with respect to surface properties.	• Determination of local minima.
Genetic	• Quick estimation; • Minimization with respect to surface properties; • Determination of global minima; • Application of non linear constrains.	• Deep knowledge of the algorithm settings is required for guaranteeing convergence of the results to the optimal solution.
Iterative	• Minimization with respect to surface properties.	• Time consuming; • Impossibility of implementing non linear constrains.

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